Table of Contents

Chapter Twelve
STATE PLANE COORDINATE SYSTEM

Chapter Twelve discusses the application of the Montana State Plane Coordinate System in highway and bridge projects and presents guidelines for employing this system during design and plan preparation. Chapter Twelve provides the designer with:

1. a discussion of the purpose of survey datums and of the vertical and horizontal control datums that are typically used by the Department;

2. information on location and survey methods that illustrates the need and benefit of employing a state plane coordinate system in highway projects;

3. a discussion of the fundamental elements of the state plane coordinate system that is adopted by the State of Montana and employed by the Department in its program of projects;

4. a discussion explaining the relationship between the ground surface, the surface of the Geodetic Reference System 1980 (i.e., GRS 80) ellipsoid and that of the state plane grid;

5. an explanation of why a distance between points on the ground differs in magnitude from the distance between the same points as they are projected onto the state plane grid; and

6. guidelines for working within the Montana State Plane Coordinate System during bridge design and plan preparation.

For additional information on survey datums, coordinate systems and related mathematical computations, the designer is referred to the Department's **Surveying Manual**.

12.1 SURVEY DATUM CONSIDERATIONS

This Section provides the designer with a discussion of the purpose of survey datums and of the vertical and horizontal control datums that are typically used by the Department.

12.1.1 Purpose of Survey Datums

A survey datum is a numeric benchmark that serves as a control reference from which other measurements can be made during design, surveying and construction. A survey control datum may be either:

1. assumed,
2. given,
3. measured, or
4. otherwise determined.

Highway and bridge projects typically require design and survey measurements to reference both a vertical and horizontal datum. Their purpose is to provide the necessary control to accurately measure and survey project elements (e.g., horizontal and vertical curves, grades, bridge decks). By referencing an accurate and well-established datum, costly mistakes can be avoided and relationships between and connections to adjoining projects can be readily established.

12.1.2 Vertical Control Datum

A vertical datum is used to accurately establish and control the elevations of and between elements in the project. Where practical, the Department references the North American Vertical Datum 1988 (i.e., NAVD 1988) for vertical control in its program of projects. When used, this vertical datum is employed consistently in both design and construction.

The datum reference will appear in both the field notes and the plan sheets. The following are cases where other vertical datum may be utilized:

1. Urban Projects. Some urban projects may require that vertical control be tied to a local, city or county datum that does not reference NAVD 1988; however, if a local datum is used, the field notes and plans will clearly state the origin of the vertical datum.

2. Small, Remote Projects. As determined on a case-by-case basis, it may be necessary to utilize an assumed datum for vertical control on small, remote projects where the expense of tying the project to NAVD 1988 is not justified. The project manager will verify the acceptability of using another datum type.

12.1.3 Horizontal Control Datum

A horizontal datum is used to accurately establish and control ground locations, lengths and angles of and between project elements during design, surveying and construction. Horizontal control is typically employed on a coordinate-based grid system, either assumed (e.g., local coordinate) or previously established (e.g., state plane). Such control is used in highway engineering to:

1. define positions of survey stations,
2. locate points-of-intersection,
3. define lines and curves, and
4. compute lengths and azimuths.

Where practical, the Department references the Montana State Plane Coordinate System (see 70-22-201 **Montana Code Annotated**). This state plane coordinate system employs the North American Datum 1983 (i.e., NAD-83) Single Zone Coordinate System. The NAD-83 system is based on computations-of-position on a precise geodetic reference ellipsoid that approximates the shape of the earth's surface at sea level. This earth-centered ellipsoid is known as the Geodetic Reference System of 1980 (i.e., GRS 80) and is established and maintained by the National Geodetic Survey using very precise, and expensive, geodetic-surveying methods.

When used, the Montana State Plane Coordinate System is employed consistently in both design and construction. The project manager will verify the acceptability of using another datum type. The datum reference and information relative to the application of the state plane coordinate system will appear in the field notes and plan sheets.

12.2 LOCATION AND SURVEY METHODS

This Section presents information on location and survey methods that illustrate the need and benefit of using a state plane coordinate system in highway projects.

12.2.1 Limitations of Plane Surveying

A conventional survey that employs plane-surveying methods and uses a horizontal control datum that is based on a local coordinate system is both cost-effective and relatively simple to implement. However, such a methods's limitations are substantial when employed in highway projects as documented in the following.

12.2.1.1 Effects of the Earth's Curvature

Plane-coordinate systems do not take into account the affects of the earth's curvature. This is typically not a major concern for surveys of limited scope; however, as horizontal control is extended from the origin of the coordinate grid, the effects of the earth's curvature become more apparent and critical.

Conventional plane-surveying methods are based on the assumption that all distances and directions are projected onto a horizontal plane that is tangent to the earth's surface at one point within the survey area. Because of the earth's curvature, as the survey departs from the point-of-tangency, there will be an increasing difference between the ground distance and the distance that results from projecting the defining points onto the coordinate grid of the horizontal plane. This deviation becomes intolerable for surveys that traverse a great distance (e.g., highway projects). When this deviation becomes intolerable, then the limits of conventional plane-surveying methods have been reached.

12.2.1.2 Relationships of Independent Surveys

Attempting to establish a relationship between multiple plane surveys, each having been conducted independently on a different horizontal datum, is both difficult and ineffective. This limitation becomes critical when attempting to connect adjoining highway projects.

If made independently, plane surveys will reference different, non-coinciding coordinate-grid systems. Furthermore, if the north-south oriented axis of each survey's coordinate system is assumed to be parallel to the true meridian (i.e., longitude) at one station within the survey, then their axes will not even be parallel due to the convergence of the meridians.

12.2.2 Positioning By Latitude and Longitude

Latitude and longitude are geographic positions that are earth-centered angular measurements. Such measurements are expressed in units of degrees, minutes and decimal seconds. Latitude is the north or south location of a point referenced from the equator. Longitude is the east or west location of a point referenced from the meridian that passes through Greenwich, England. For example, Beartooth Pass, Montana is located at approximately 45° 00′ 17″ North Latitude and 109° 24′ 36″ West Longitude. While positioning by latitude and longitude is useful for navigation, it is not practical nor convenient to employ as a method-of-location in highway engineering work.

12.2.3 Benefits of Geodetic Surveying

In the methods of geodetic surveying, all distances are reduced to a common reference surface that conforms closely to sea level. Coordinates of points are computed with reference to parallels of latitude and meridians of longitude by using angles computed near the center of the earth rather than distances.

Geodetic surveying is employed so that precise surveys may be extended over great distances in any direction without suffering the limitations of plane-surveying methods; however, geodetic-surveying methods are:

1. more complex and more expensive,
2. involve more difficult computations, and
3. require specialized personnel in execution.

The National Geodetic Survey has established and maintains horizontal control monuments in the form of triangulation, traverse and intersection stations. These stations have been located by the methods of geodetic surveying. The control points in this network are continually being updated and bear a definite relationship, one to another, being referred to one common reference surface.

12.2.4 State Plane Coordinate Systems

State plane coordinate systems are made possible by virtue of the highly precise geodetic-surveying methods used in establishing the initial framework of the horizontal control that defines these systems. This horizontal control can be readily incorporated into conventional surveys for the purposes of:

1. coordinating,
2. checking,
3. establishing, and
4. reestablishing survey points.

Employing a state plane coordinate system allows the relatively simple methods of plane surveying to be used over great distances in any direction while accounting for the affects of the earth's curvature and maintaining a precision approaching that of geodetic surveying. For the surveys that are tied to a state plane coordinate system, each will be related to one another. This is the most singular advantage of a state plane coordinate system. Other advantages of using a state plane coordinate system are as follows:

1. Greater Distances. The use of a state plane coordinate system permits conventional surveys to be carried over Statewide distances by using plane-surveying methods with results that approach that obtained by geodetic-surveying methods.

2. Project Conformance. Plans which have been controlled by coordinated points on a state plane will always conform when joined, no matter how unrelated the projects were that necessitated the plans.

3. Consistent Basis of Bearing. By tying a project to a state plane coordinate system, especially where multiple surveys are involved, a consistent basis of bearing may be established.

4. GIS Project Database. By employing a state plane coordinate system, a Geographic Information System (i.e., GIS) database of projects can be established and maintained for an entire state whether the projects are conducted by a state or a local government agency.

5. Consistent Use of GPS. The deployment of the satellite-based Global Positioning System (i.e., GPS) greatly simplifies obtaining known positions for conversion to State plane coordinates. GPS allow the expanded use of State plane coordinates in projects.

6. Increase in Accuracy. A traverse of relatively low accuracy that is run between a pair of control points is actually raised in accuracy after an adjustment between the control points is made.

7. Reduction in Error. The use of well-established control points in a traverse eliminates serious errors in measuring both distances and angles.

8. Fewer Control Surveys. The use of a common reference system reduces or eliminates the costly duplication of control surveys possibly required by multiple projects in the same area.

9. Reestablishment of Lost Points. A point whose coordinates have been determined

can, if lost, always be replaced with the degree of precision in which it was originally established

10. Aerial Control/Mapping. Photogrammetric mapping can be conducted at much less expense when all control points in the area to be mapped are on the same system. State plane coordinates are shown on many federal maps, particulary United States Geological Survey Quadrangle Topographic Maps.

12.3 THE MONTANA STATE PLANE COORDINATE SYSTEM

This Section discusses the fundamental elements of the state plane coordinate system used by the State of Montana and the Department.

12.3.1 Montana State Statute

The Montana State Legislature has enacted laws establishing the legal statute of the Montana State Plane Coordinate System. The designer is referred to §70-22-201 of the **Montana Code Annotated** for a definition of the Montana State Plane Coordinate System, its use and limitations.

12.3.2 Geodetic Reference Datum

The geodetic reference system used for the Montana State Plane Coordinate System is the Geodetic Reference System of 1980 (i.e., GRS 1980). This reference system is maintained by the National Geodetic Survey using the very precise methods of geodetic surveying. GRS 80 is an earth-centered ellipsoid that approximates the shape of the earth's surface at sea level. The computations-of-positions on this ellipsoid are known as the North American Datum of 1983 (i.e., NAD-83). The State of Montana has elected to develop its state plane coordinate system based on the NAD-83 Single Zone Coordinate System.

The National Geodetic Survey publishes NAD-83 coordinates in the metric system-of-units (i.e., meters). The Department publishes NAD-83 coordinates in either meters or positions of latitude and longitude. The Montana State Plane Coordinate System specifies that the conversion factor that should be used to convert between the English and metric systems is the international conversion factor of 1 ft = 0.3048 m.

12.3.3 Lambert Conformal Projection

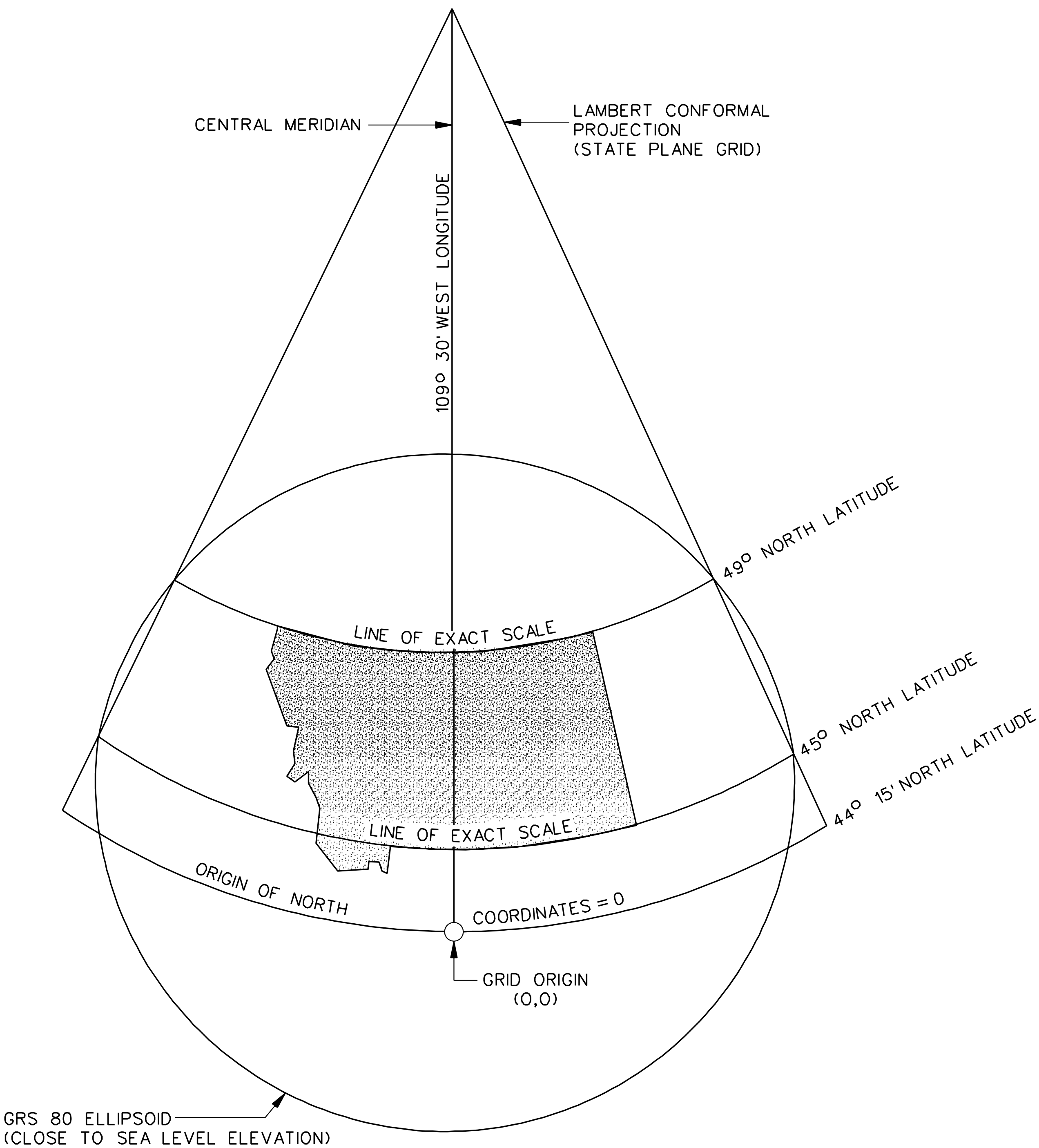
The Montana State Plane Coordinate System is based on the NAD-83 Single Zone Coordinate System and is developed through use of the Lambert conformal projection as represented diagrammatically in Figure 12.3A. For Montana, the NAD-83 point-of-origin is defined as 44° 15′ North Latitude and 109° 30′ West Longitude (i.e., the State's central meridian). The projection employs a conical surface with its axis coinciding with that of the earth's rotation. The GRS 80 ellipsoid (i.e., that surface approximating the earth's surface at sea level) is intersected by the cone along two parallels of latitude known as the lines of exact scale (i.e., 49° and 45° North Latitude). These lines are approximately equidistant from a parallel lying in the center of the State. The state plane grid is actually developed by mathematically projecting positions of latitude and longitude from the GRS 80 ellipsoid onto the conical surface (i.e., the state plane). Montana uses the Lambert conformal projection as distortions occur over the State's relatively short north-south direction.

Along the lines of exact scale, distances on the state plane grid are the same as corresponding distances on the GRS 80 ellipsoid. Between lines of exact scale, a distance on the state plane grid is smaller than the corresponding distance on the GRS 80 ellipsoid. Outside the lines of exact scale, a projected distance is larger. The discrepancy between corresponding distances depends on the position of the line being considered with respect to the lines of exact scale as follows:

- The scale of a line running in a north-south direction varies from point-to-point.
- A due east-west line has a constant scale along its length, whether it be larger than, equal to or less than that scale corresponding to the GRS 80 ellipsoid.

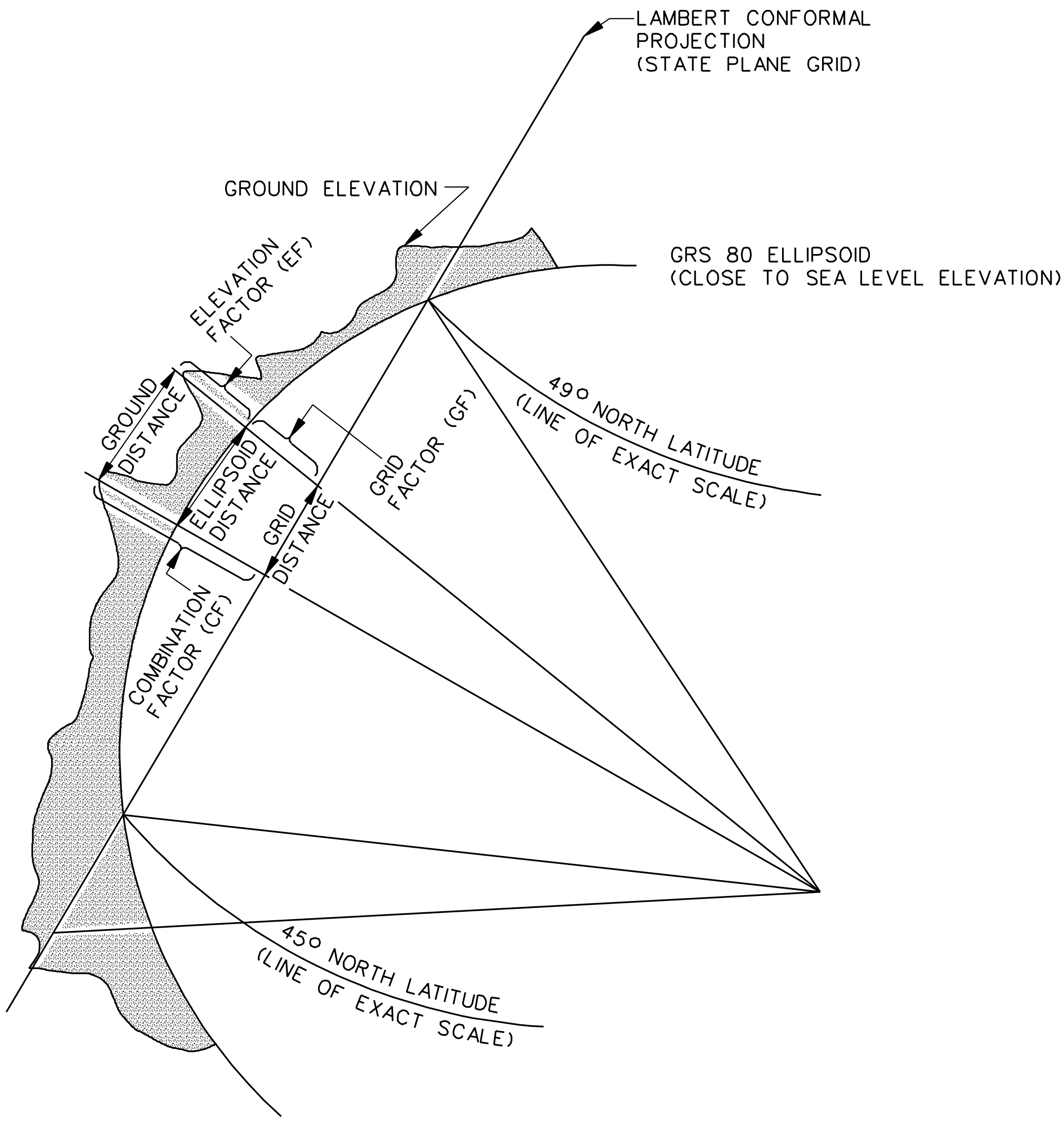
12.3.4 Projections on the State Plane Grid

Figure 12.3B presents a diagram that illustrates the relationship of a distance as it appears during projection on:



THE LAMBERT CONFORMAL PROJECTION OF THE MONTANA STATE PLANE GRID

Figure 12.3A



**RELATIONSHIP BETWEEN THE GROUND SURFACE,
THE GRS 80 ELLIPSOID AND THE STATE PLANE GRID**

Figure 12.3B

- the ground surface;
- the surface of the Geodetic Reference System 1980 (i.e., GRS 80) ellipsoid; and
- the surface of the state plane grid.

As illustrated by Figure 12.3B, a distance between points on the ground will differ in magnitude from the distance between the same points as they are projected onto the state plane grid. This fact necessitates the use of scaling factors to convert a corresponding distance as it is projected through the three surfaces. The following Section defines the scaling factors that are typically employed (i.e., grid, elevation and combination factors).

12.3.4.1 Grid Factor

The grid factor (GF) is a dimensionless scale, or multiplication, factor that is used to convert a distance between points on the GRS 80 ellipsoid (i.e., at approximately sea level) to an equivalent distance between the points as they are projected onto the state plane grid (i.e., on the Lambert conformal projection). The following defines the ranges in magnitude of the grid factor that can be expected in Montana:

1. <u>GF < 1.0</u>. At locations between lines of exact scale (i.e., between 49° North and 45° North Latitudes), the grid factor is less than 1.0. This region encompasses the majority of locations in the State.
2. <u>GF = 1.0</u>. The grid factor is equal to 1.0 at locations along lines of exact scale (i.e., along either 49° North or 45° North Latitudes).
3. <u>GF > 1.0</u>. At locations below 45° North Latitude (i.e., the southwest tip of the State), the grid factor is greater than 1.0.

12.3.4.2 Elevation Factor

The elevation factor (EF) is a dimensionless scale, or multiplication, factor that is used to convert a distance between points on the ground to an equivalent distance between the points as they are projected onto the GRS 80 ellipsoid (i.e., at approximately sea level). The following defines the ranges in magnitude of the elevation factor:

1. <u>EF < 1.0</u>. The elevation factor is less than 1.0 at locations that are above sea level elevation (i.e., outside the GRS 80 ellipsoid).
2. <u>EF = 1.0</u>. For locations that are at approximately sea level (i.e., on the GRS 80 ellipsoid), the elevation factor is equal to 1.0.
3. <u>EF > 1.0</u>. At locations that are below sea level (i.e., inside the GRS 80 ellipsoid), the elevation factor is greater than 1.0.

12.3.4.3 Combination Factor

The combination factor is a dimensionless scale, or multiplication, factor that is used to convert a distance between points on the ground to an equivalent distance between the points as they are projected onto the state plane grid (i.e., on the Lambert conformal projection). By definition, the combination factor for a particular location is the product of the location's grid and elevation factors. The following equation represents the mathematical relationship between the combination, grid and elevation factors:

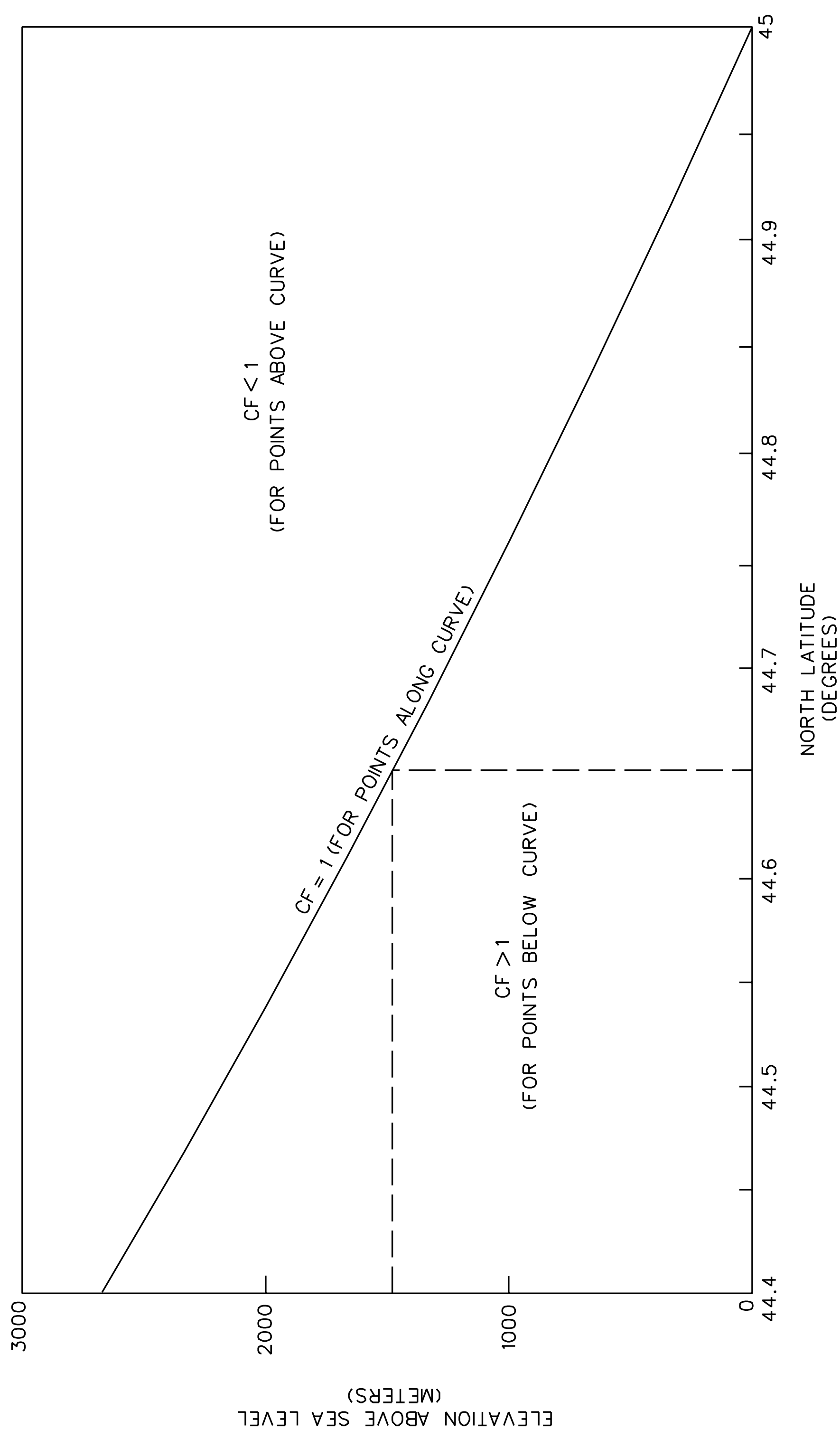
$$CF = (GF)(EF)$$

The range in magnitude of the combination factor at most locations in Montana (i.e., between 49° North and 45° North Latitudes) is less than 1.00000000 and greater than 0.99900000. A possible exception to this is that the combination factor may be greater than 1.0 at locations in the southwest tip of the State (i.e., below 45° North Latitude). In this region, the grid factor is greater than 1.0. If the elevation factor closely approximates 1.0, then the product of the grid and elevation factors may result in a combination factor that is greater than 1.0. Figure 12.3C provides a graph that illustrates this relationship. For example, at a location near 44.65° North Latitude that has an elevation of approximately 1500 m, the combination factor will approximate 1.0. Figure 12.3D presents a table that illustrates how the grid, elevation and combination factors vary according to location.

Location	Avon, MT	Lindsay, MT	Beartooth Pass, MT	Troy, MT	Monida Pass, MT
North Latitude	46° 35′54″	47° 13′13″	45° 00′17″	48° 27′52″	44° 33′20″
West Longitude	112° 35′31″	105° 09′02″	109° 24′36″	115° 53′44″	112° 16′00″
Elevation, m	1431.00	817.00	3121.00	575.00	2094.00
Grid Factor (GF)	0.999418616	0.999399187	0.999997169	0.999715546	1.000295660
Elevation Factor (EF)	0.9997754742	09998717992	0.9995104407	0.9999097696	0.9996714827
Combination Factor (CF)	0.9991942207	0.9992710633	0.9995076111	0.9996253413	0.9999670456

GRID, ELEVATION AND COMBINATION FACTORS FOR VARIOUS LOCATIONS IN MONTANA

Figure 12.3D



RELATIONSHIP BETWEEN ELEVATION AND COMBINATION FACTOR FOR LOCATIONS BELOW 45E NORTH LATITUDE

Figure 12.3C

12.4 GUIDELINES FOR BRIDGE DESIGN AND PLAN PREPARATION

This Section provides the designer with guidelines for working within the Montana State Plane Coordinate System during bridge design and plan preparation.

12.4.1 Survey Stationing and Plan Dimensions

In general, survey stationing for the Department's projects is employed on the state plane. However, the horizontal dimensions that are shown on the plans reflect actual ground dimensions. Project elevations are computed based on both state plane stationing and the vertical curve data provided by the Road Design Section. Although horizontal angles on the ground correspond to those on the state plane, lengths and areas require the application of the combination factor.

12.4.2 Application of the Combination Factor

The Department's projects are established with reference to control points on the state plane, and their construction plans and drawings contain the state plane coordinates and grid lines. The state plane coordinates of two ends of a construction line that appears on a plan may be used to compute the line's grid length and grid bearing or azimuth; however, the proper length to be used in design and construction is the line's actual length on the ground. The conversion between grid length and ground length can be achieved by employing the combination factor in one of two ways as follows:

1. Ground Distance to Grid Distance. To convert a ground distance to an equivalent distance on the state plane grid, multiply the ground distance by the combination factor.

2. Grid Distance to Ground Distance. To convert a distance on the state plane grid to an equivalent distance as measured on the ground, compute the length between the state plane coordinates and divide the resultant grid distance by the combination factor.

If the state plane coordinates of the corners of an area are known, the grid area encompassed within the defining boundary may be computed. Because this grid area does not correspond to the actual ground area, the difference is taken into account by employing the combination factor as follows:

1. Ground Area to Grid Area. To convert a ground area to an equivalent area on the state plane grid, multiply the ground area by the square of the combination factor.

2. Grid Area to Ground Area. To convert an area that is on the state plane grid to an equivalent area as measured on the ground, compute the area bounded by the state plane coordinates and divide the resultant grid area by the square of the combination factor.

To accommodate proper application of the combination factor without misunderstanding, a note is generally placed on each drawing within the plan set. This note gives explicit instructions to the user as illustrated by the following example:

Note: All distances shown on this plan are grid distances on the Montana State Plane Coordinate System. To obtain ground distances for laying out construction lines, divide grid distances by the combination factor 0.9998940.

The Photogrammetry and Survey Section provides the combination factor that is used in the note. The magnitude of the factor may differ among plan sheets depending on the length, orientation and location of the project as it is designed to convert between grid and ground distances for specific geographic locations.

12.4.3 Preliminary General Layout

The combination factor may be ignored in all preliminary engineering calculations leading up to the preliminary general layout. At this point, all survey and mapping data should be on the state plane. The Hydraulics Section typically ignores the use of the combination factor in their analyses, and intersections and topography are referenced to state plane stationing.

Once the preliminary general layout has been determined, the method for preparing the final general layout will depend on the existence of site constraints at the substructure location. For when to apply the combination factor during the preparation of the final general layout, substructure locations may be categorized as either critical or non-critical as discussed in the Sections that follow.

12.4.4 Critical Substructure Locations

Critical substructure locations are where the substructure unit requires a precise location. Examples include urban jobs and railroad crossings where an accurate location of the substructure is necessary to accommodate the specific site and meet its dimensional and clearance requirements.

Precise locations of critical substructure stations are typically determined during preliminary engineering. During the preparation of the final general layout for a critical substructure unit, the following steps should be performed:

1. Fix the critical substructure stations that were determined during preliminary engineering.

2. Recompute actual span lengths by applying the combination factor.

12.4.5 Non-Critical Substructure Locations

Non-critical substructure locations are where the substructure unit does not require a precise location. Water crossings, as well as some urban and railroad crossings, are typically categorized as non-critical. Small deviations (i.e., less than 0.1 m) in the location of a non-critical substructure unit will generally have no significant adverse affect on the overall project.

During the preparation of the final general layout for a non-critical substructure unit, the following steps should be performed:

1. Fix the span lengths to a convenient number (e.g., multiples of 0.500 m).

2. Recompute the non-critical substructure stations using the combination factor.

3. Select one reference station to use for the structure. This is typically either the middle or one end of the structure.

4. Compute all non-critical substructure stations referenced from the fixed reference station by applying the combination factor to the actual span lengths.

12.4.6 Final General Layout and Footing Plan

The following guidelines are applicable to the development of the final general layout and footing plan:

1. Project Stationing. Compute and present all project stations as state plane stationing.

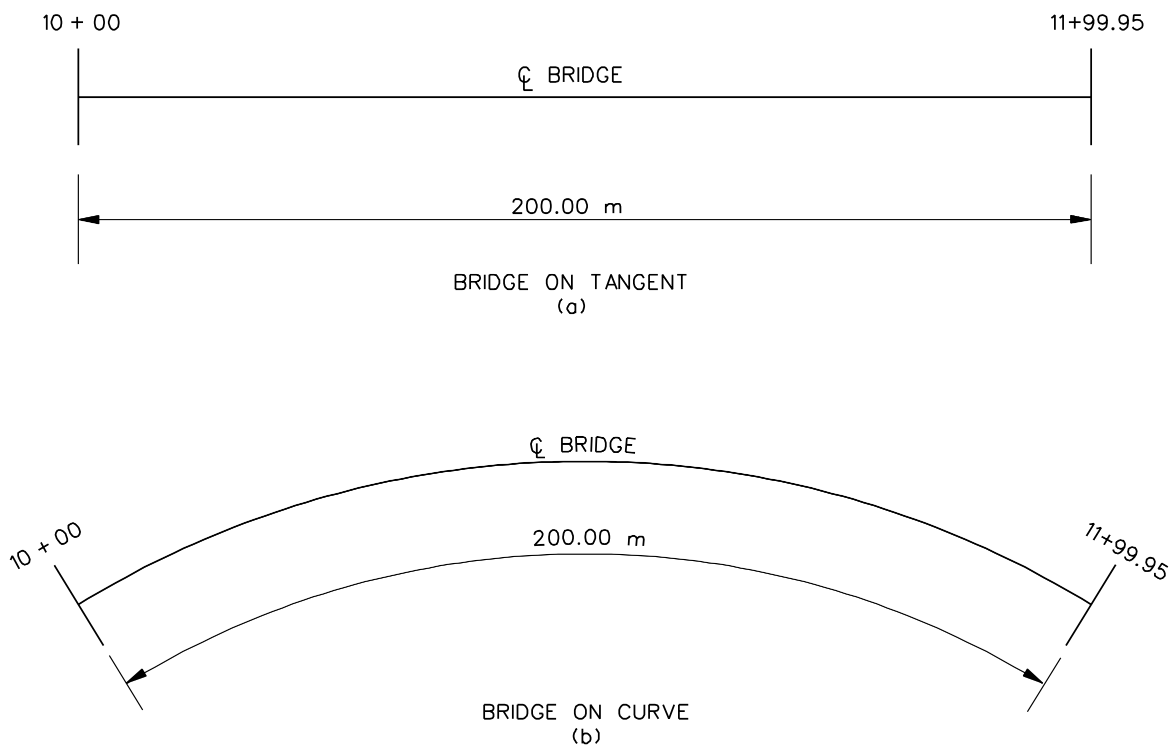
2. Curve Data. The vertical and horizontal curve data provided by the Road Design Section will be state plane distances and will be consistent with the data on the highway plans.

3. Structural Dimensions. Compute and present all dimensioned distances as actual ground distances, not distances on the state plane.

4. State Plane Coordinate System Note. Include the standard state plane coordinate system note on the final general layout. This note gives the combination factor that is applicable to the structure's site. See Section 12.4.2.

5. Total Length of Structure. Compute and present the total length of the structure along its centerline between the end stations. This

dimension shows that the actual ground length differs from the length of the structure as computed from the difference between its end stations (i.e., its length on the state plane grid). This is diagrammatically illustrated in Figure 12.4A.



PRESENTATION OF STRUCTURE LENGTH AND END STATIONS ON FINAL GENERAL LAYOUT

Figure 12.4A